Abstract—Signal space diversity (SSD) is a well-known technique providing excellent performance gain over fading channels. This paper investigates the system design of bit-interleaved coded modulation with iterative demodulation and signal space diversity (BICM-ID-SSD). It demonstrates that within SSD the average mutual information (AMI) between the signal after rotated constellation mapping and that before the soft demapper varies with the rotation angle. Therefore, a new criterion for searching the optimal rotation angle by maximizing such AMI is proposed. Based on this criterion it is shown that the optimal rotation angle is not relevant to the labeling, and 45-degree is found to be the optimal or near optimal rotation angle for square quadrature amplitude modulation (QAM) at low to moderate code rates. Furthermore, the procedure of BICM-ID-SSD system design could be divided into two independent steps: 1) determining the optimal rotation angle based on the above criterion, and 2) choosing well-fitted labeling and outer channel code with the aid of extrinsic information transfer (EXIT) charts. Analysis and simulation results show that the proposed system exhibits a near-capacity performance, meanwhile being robust over both additive white Gaussian noise (AWGN) and Rayleigh fading channels.

I. INTRODUCTION

Originally proposed by J. Boutros and E. Viterbo [1], signal space diversity (SSD) is a well-known technology that provides significant diversity gain over fading channels without any power or bandwidth penalty over additive white Gaussian noise (AWGN) channel. The basic idea of SSD is to use coordinate interleaving, also named the in-phase and quadrature phase (I/Q) interleaving for two dimensional constellations, and together with constellation rotation to increase the diversity order. Determining the optimal constellation rotation angle in SSD is an open problem.

Many papers tried to solve this problem, which can be classified mainly into two categories: rotation for uncoded systems [1]–[4] and for coded systems. Since practical systems always require channel coding, optimization for coded system is of more importance. The most promising coded modulation schemes over fading channels are bit-interleaved coded modulation (BICM) [5] and its iterative version, BICM with iterative demapping and decoding (BICM-ID) [6], [7]. The rotation problem for BICM systems is discussed in [8] and [9], and that for BICM-ID is extensively addressed in [10]–[14]. These papers try to find the optimal rotation angle either based on the BER bound [13], [14], or via computer simulation [11], [12]. Nevertheless, computer simulation costs too much time and the results can not be generalized, because they are relevant to a plurality of specific factors such as the channel code and labeling. BER bound is a good criterion, but it also depends on these factors, and thus makes the optimization problem complicated. Moreover, BER bound is not suitable for noisy channels with low or moderate signal to noise ratio (SNR), and therefore can not provide useful guidelines for Shannon-limit approaching coded modulation system (CMS) design.

Information theory established fundamental limits for all CMSs. It is shown in this paper that within SSD the average mutual information (AMI) between the signal after rotated constellation mapping and the signal before the soft demapper varies with the rotation angle over fading channels, for the case with perfect channel state information available at the receiver (CSIR). Therefore, maximizing such AMI is proposed as the criterion to determine the optimal rotation angle.

This paper is organized as follows. The SSD system model is first described in Section II. Section III deals with the optimal constellation rotation problem for BICM-ID systems with SSD (BICM-ID-SSD). Extrinsic information transfer (EXIT) chart [15] is employed in Section IV to help BICM-ID-SSD system design. BICM-ID-SSD systems with 16/64 quadrature amplitude modulation (QAM) are detailed, where the technique of doping [16] is also employed for error-floor removal. BER simulations are carried out in Section V to demonstrate the efficiency of the AMI and EXIT chart analysis, as well as to show the excellent performance of the proposed system. Conclusions are drawn in Section VI.

II. SYSTEM MODEL

The system model of SSD over fading channels is depicted in Fig. 1. As shown in this figure, an m-tuple encoded bit vector \( B = [B_0B_1 \ldots B_{m-1}]^T \) is mapped on to a constellation symbol \( X \), where the superscript \(^T\) denotes transposition. The I/Q components of the mapped symbols are then interleaved, and new symbols are reconstructed based on these interleaved I/Q components and transmitted over the fading channels. With perfect CSIR, the received signal is first phase-equalized and then I/Q de-interleaved before sending to the soft demapper. Assuming that the interleaver is long and random enough, each component of the symbol sent to the demapper can be regarded as suffering from independent fading and be modeled as

\[
Y_K = \Lambda_K X_K + N_K, K \in \{I, Q\}
\]
the size of the signal set. The channel input \( \Lambda \) of all systems, the a priori knowledge of each bit is fed back from the receiver. Usually, the log-likelihood ratio (LLR) \( Y \approx N \text{i.i.d. fading coefficients.} \)

\[ Y \equiv [X_I, X_Q]^T, N = [N_I, N_Q]^T \text{ and } \Lambda = \text{diag}(\Lambda_I, \Lambda_Q) \]

Rayleigh fading channels, \( \Lambda \) are identically independently distributed (i.i.d.) fading coefficients. \( N_I \) and \( N_Q \) are i.i.d. Gaussian random noise with zero mean and variance of \( N_0/2 \). \( X_I \) and \( X_Q \) are the I/Q components of the rotated symbol. For Rayleigh fading channels, \( \Lambda_I \) and \( \Lambda_Q \) are i.i.d. normalized Rayleigh distributed with the probability density function of

\[ p(\lambda) = 2\lambda \exp(-\lambda^2), \lambda \geq 0. \]

By denoting \( X = [X_I, X_Q]^T, Y = [Y_I, Y_Q]^T, N = [N_I, N_Q]^T \text{ and } \Lambda = \text{diag}(\Lambda_I, \Lambda_Q) \) to represent a 2 \times 2 diagonal matrix with the diagonal elements \( \Lambda_I \) and \( \Lambda_Q \), the channel modeled by (1) can also be written in the matrix form as \( Y = \Lambda X + N \). Based on the symbol \( Y \) and the CSIR \( \Lambda \), usually the log-likelihood ratio (LLR) of all \( m \) bits \( L = [L_0, L_1 \cdots L_{m-1}] \), can be calculated by the soft demapper and sent to the channel decoder. For BICM-ID systems, the a priori knowledge of each bit is fed back from the decoder to the demapper and iterative process is accordingly performed.

III. MUTUAL INFORMATION ANALYSIS

A. Mutual Information Elements

Channel capacity offers a tight upper bound for all CMSs with reliable transmission. However, usually the capacity can only be achieved via a particular input distribution. For instance, the capacity of power-limited AWGN channel can only be achieved via Gaussian input, which is not practical in real systems. Actually, the channel input is always constrained by a specific constellation for digital communication systems, and therefore such constellation constrained AMI becomes the new upper bound, which is called the constellation constrained capacity in [17], and named coded modulation AMI (CM-AMI) in this paper. An optimal CMS such as BICM-ID should have the potential to approach such CM-AMI. To design a good BICM-ID system, one important task is to maximize the CM-AMI.

As the channel is modeled by (1), CM-AMI with perfect CSIR, can be written and evaluated as

\[ I_{CM} = I(X; Y|\Lambda) = m - E_{x,y,\Lambda} \left[ \log_2 \sum_{\chi \in \chi} p(y|\hat{x},\Lambda) \right] p(y|x,\Lambda), \]

where \( I(\cdot;\cdot) \) denotes the AMI function, \( \chi \) denotes the rotated signal constellation set and \( m = \log_2 |\chi| \) whereby \( |\chi| \) denotes the size of the signal set. The channel input \( X \) is constrained by the signal set \( \chi \). Different rotation angles result in different signal sets and consequently different CM-AMI.

B. Some Properties

1) Labeling: Most labeling functions are one-to-one functions, i.e., the \( m \)-bit sequence \( B \) \( \Rightarrow \) the constellation point \( X \). Thereby, apparently the following function holds that

\[ I(B; Y|\Lambda) = I(X; Y|\Lambda). \]

Since \( I(X; Y|\Lambda) \) is independent with the labeling function, it indicates that for BICM-ID-SSD systems, CM-AMI has nothing to do with labeling even from the viewpoint at bit-level. Therefore, the optimal constellation rotation angle is also independent with the labeling, let along the outer channel code, for BICM-ID-SSD systems. This property allows us to divide the overall procedure of BICM-ID-SSD design into two independent phases, i.e., determine the optimal rotation angle first, and then obtain a well-fitted pair of labeling and outer code.

2) Square QAM without rotation: For any square \( M \)-QAM without rotation, \( X_I \) and \( X_Q \) are independent with each other, i.e., \( \Pr \{X_I = x_I, X_Q = x_Q\} = \Pr \{X_I = x_I\} \Pr \{X_Q = x_Q\} \). Thereby, the I/Q channels are independent with each other and that

\[ I(X; Y|\Lambda) = I(X_I; Y_I|\Lambda_I) + I(X_Q; Y_Q|\Lambda_Q), \]

which indicates that the CM-AMI will be unchanged even if I/Q interleaving is employed for square QAM without rotation.

3) Without I/Q interleaving or over AWGN channel: Without I/Q interleaving, \( \Lambda_I \equiv \Lambda_Q \). Over AWGN channel, \( \Lambda_I \equiv \Lambda_Q \equiv 1 \). Thereby, AWGN channel can be regarded as a special case of those without I/Q interleaving, under which condition the received signal \( Y \) can be rotated back to traditional signal, and the noise distribution remains unchanged. As a result, the CM-AMI will keep the same if only constellation rotation is used or over AWGN channel.

C. Numeric Results

Given a constellation and a fading channel, the CM-AMI is a function of the rotation angle \( \theta \) and the SNR, i.e., \( I_{CM}(\theta, \text{SNR}) \). We first fix the SNR and plot how \( I_{CM}(\theta, \text{SNR}) \) varies with the rotation angle \( \theta \). Due to the symmetric of square QAMs, we apparently have \( I_{CM}(90^\circ - \theta, \text{SNR}) = I_{CM}(\theta, \text{SNR}) \). Therefore, the rotation range is set as \([0, 45^\circ]\).

The numeric results are shown in Fig. 2 for 4QAM, Fig. 3(a) for 16QAM and Fig. 3(b) for 64QAM, respectively, over Rayleigh fading channels. As shown in these figures, the rotation angle \( \theta = 45^\circ \) is a good choice for all the 4/16/64QAM within a large SNR scale. For example, it can be regarded optimal or near optimal at both SNR = 1.5 dB and SNR = 6 dB for 4QAM, around which the CM-AMIs are about 1 bit/channel use and 1.5 bits/channel use corresponding to 1/2 and 3/4 code rates, respectively.

We now fix the rotation angle as \( 45^\circ \) and plot how \( I_{CM}(\theta, \text{SNR}) \) varies with SNR. The X-axis is chosen as the AMI and the Y-axis as the gap to the Gaussian input, by which distribution the channel capacity with CSIR can be achieved as \( C = E_{\Lambda} \log_2 (1 + \lambda^2 \text{SNR}) \). The numeric results are shown...
in Fig. 4, based on which the constellation rotation gain over Rayleigh fading channel can be easily observed. For instance, for 1/2 code rate 16QAM, the rotation gain with the angle of 45° is about 0.2 dB, and such gain becomes larger as the code rate increases. Moreover, it also indicates that, to achieve a given spectrum efficiency for a particular CMS, a high-order-constellation together with a low- or moderate-rate-code is better than a low-order-constellation with a high-rate-code.

IV. EXIT CHART ANALYSIS

A. SSD Effect to the EXIT Demapper Curve

EXIT chart is a powerful tool to analyze the convergence behavior of iterative systems [15]. It is well-known that curve-fitting in EXIT charts is an essential way for BICM-ID system design, i.e., to choose a pair of well-fitted labeling and outer channel code from the EXIT chart point of view. An EXIT chart example for 16QAM demapper is shown in Fig. 5. In this figure, the demapper curves exhibit different shapes under different channels. The demapper curve under Rayleigh fading channel (the dot-dashed line) has a smaller slope than that under AWGN channel (the dashed line) in traditional systems without I/Q interleaving, as shown in Fig. 5 with a typical random labeling. Therefore, a well-fitted pair of labeling and outer code under AWGN channel may not performs well enough under fading channels. Fortunately, I/Q interleaving can shape the demapper curve a larger slope in fading channels, even without constellation rotation for most labelings. Labelings with all their binary signal sets lie vertically or horizontally without constellation rotation, such as Gray labeling, do not benefit from this property.

An important property of EXIT chart is that the area under the demapper curve $A_{dem}$ approximately equals to the CM-AMI over the number of bits per constellation point $m$, $A_{dem} \approx I_{CM}/m$ [18]. Since the CM-AMI keeps constant without rotation for square QAMs as discussed in Section III, it is interesting that without rotation, the areas under the demapper curves with/without I/Q interleaving appear the same, and such areas become larger if rotation is further employed, as shown in Fig. 5.

B. BICM-ID-SSD System Design Based on EXIT Charts

The technique of doping [16], also named the unit-rate precoding [19], is employed to avoid the error-floor. The system model of BICM-ID-SSD with doping is shown in Fig. 6. The doping code used is a unit-rate 2-state recursive systematic convolutional (RSC) code, whose encoder is depicted in Fig. 6(b) with every $P$th information bit being replaced by a coded bit, where $P$ is called the doping rate.

Labelings are well-known crucial for BICM-ID. An improved algorithm based on binary switch algorithm (BSA) [20] is employed here for searching them. The labelings for 16QAM and 64QAM are depicted in Fig. 7, and the doping
rate is set as $P = 100$. A rate-1/2 4-state non-recursive convolutional (NRC) [7, 5] code is chosen as the outer code. The compact notation NRC($[G_1, G_2]_8$) denotes a NRC code with feed forward polynomials $G_1$ and $G_2$ in octal. The outer code uses the standard BCJR decoding algorithm, and the concatenation of the doping code and the demapper uses the so-called serial detection method [16].

The EXIT charts are provided in Fig. 8 and Fig. 9 for 16QAM and 64QAM, respectively. Based on these two figures, it can be concluded that the labelings shown in Fig. 7 fit the outer channel code very well, over both AWGN and Rayleigh fading channels when SSD is employed. Therefore, the proposed BICM-ID-SSD systems would exhibit excellent performance over both channels.

V. SIMULATION RESULTS

Several parameters of the proposed BICM-ID-SSD system have already been described in Section IV. Others are listed as follows. The block length is set as 64,800 bits, wherein an $S$-random interleaver with $S = 100$ is chosen as the bit-wise interleaver [21]. The system takes 100 iterations.

The BER performance over Rayleigh fading channel is shown in Fig. 10 and Fig. 11 for 16/64QAM, respectively. As shown in these two figures, the proposed systems are only about 0.7 to 0.8 dB away from the CM limits at BER of $10^{-5}$, either for 16 or 64QAM, either with or without rotation. The rotation gains are about 0.3/0.2 dB for 16/64QAM at rate 1/2, which matches well with the AMI analysis results shown in Fig. 4. Moreover, traditional BICM-ID systems without SSD display much worse performance than the proposed BICM-ID-SSD systems. The reason is that the demapper curves of traditional BICM-ID do not fit well the outer-channel-code curve, as shown in Fig. 8 and Fig. 9 in order to maintain a well-fitted pair over AWGN channel. Therefore, SSD plays a very important role for a BICM-ID-SSD system to exhibit excellent performance over both AWGN and fading channels.

VI. CONCLUSIONS

This paper investigated the problem of BICM-ID-SSD system design. A new criterion for determining the optimal rotation angle in SSD via maximizing the CM-AMI is proposed,
AWGN and fading channels. ID-SSD systems achieve excellent performance under both can be mitigated by SSD, which makes the proposed BICM-ID-SSD systems exhibit excellent performance, i.e., they are about only 0.7 to 0.8 dB away from the CM limits. Moreover, it is shown the problem that the EXIT demapper curves under Rayleigh fading channels exhibit different slopes is shown the problem that the EXIT demapper curves under Rayleigh fading channels. After obtaining the optimal rotation angle, the tool of EXIT chart is used to help the BICM-ID-SSD system based on which it is shown that the optimal rotation angle is not relevant to the labelings or the outer channel codes, and the optimization problem is therefore significantly simplified. The rotation angle of $45^\circ$ is found optimal or near-optimal for square QAM at low to moderate code rates, over Rayleigh fading channels. After obtaining the optimal rotation angle, the tool of EXIT chart is used to help the BICM-ID-SSD system design. Simulation results show that the proposed BICM-ID-SSD systems exhibit excellent performance, i.e., they are about only 0.7 to 0.8 dB away from the CM limits. Moreover, it is shown the problem that the EXIT demapper curves under AWGN and Rayleigh fading channels exhibit different slopes can be mitigated by SSD, which makes the proposed BICM-ID-SSD systems achieve excellent performance under both AWGN and fading channels.

Fig. 10. BER of the proposed BICM-ID-SSD system with/without constellation rotation, 16QAM i.i.d. Rayleigh fading channels.

Fig. 11. BER of the proposed BICM-ID-SSD system with/without constellation rotation, 64QAM i.i.d. Rayleigh fading channels.

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References


